SI Units and Quantity Calculus for Conversions*

The quantity calculus is one of the easiest methods for converting the units of a physical quantity. A physical quantity consists of two parts, the numerical value and the unit.

physical quantity = numerical value × unit 1

For an example, if a distance (wavelength) for a physical quantity whose symbol is λ is given by:

$\lambda = 5.869 \times 10^{-7}$ m 2

5.869×10^{-7} is the numerical value and m (meters) is the unit. In SI, units consist of two parts. One part is the SI base or derived unit. The other part is the prefix which is used to modify the numerical value.

For example, m as a prefix unit means 10^{-3}. Therefore:

2.0×10^{-3} g $\equiv$ 2.0 mg 3

The prefix n means 10^{-9}, therefore, for the physical quantity λ above:

$\lambda = 5.869 \times 10^{-7}$ m $\equiv$ 586.9 nm 4

The following table is the accepted SI prefixes. The ones which are needed for General Chemistry, which you should learn, are in bold.

SI unit prefixes

Value	name	symbol	Value	name	symbol
x 10^{-1}	deci	d	x 10^{1}	deca	da
x 10^{-2}	**centi**	**c**	x 10^{2}	hecto	h
x 10^{-3}	**milli**	**m**	**x 10^{3}**	**kilo**	**k**
x 10^{-6}	**micro**	**μ**	**x 10^{6}**	**mega**	**M**
x 10^{-9}	**nano**	**n**	x 10^{9}	giga	G
x 10^{-12}	pico	p	x 10^{12}	tera	T
x 10^{-15}	femto	f	x 10^{15}	peta	P
x 10^{-18}	atto	a	x 10^{18}	exa	E
x 10^{-21}	zepto	z	x 10^{21}	zetta	Z
x 10^{-24}	yocto	y	x 10^{24}	yotta	Y

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* This discussion follows the June 1992 IUPAC/IUPAP conventions. See the IUPAC publication Quantities, Units and Symbols in Physical Chemistry, I. Mill, T. Cvitas, K. Homann, N. Kallay and K Kuchitsu, Blackwell Scientific Publications, Oxford (1993).

The SI base units are according to the following table. The base units you will use in General Chemistry are bolded. Included on this table is the quantity or algebraic symbol. The quantity symbol is used to represent the physical quantity in an equation and need not be explained as to what it is, unless there is a conflict. If any other symbol is used, it must be defined.

SI base units

Physical Quantity	name	unit symbol	quantity symbol
length	**meter**	**m**	***l, x, y, z***
mass	**kilogram**	**kg**[#]	***m***
time	**seconds**	**s**	***t***
electric current	**ampere**	**A**	***I, i***
temperature	**Kelvin**	**K**	***T***
amount of substance	**mole**	**mol**	***n***
luminous intensity	candela	cd	I_v

There and many SI derived units. Listed below are some which will be useful in General Chemistry.

Some SI derive units

Physical Quantity	name	unit symbol	derived from:	quantity symbol
frequency	Hertz	Hz	s^{-1}	ν, f
pressure	Pascal	Pa	m^{-1} kg s^{-2}	p, P
energy, work, heat	Joule	J	m^2 kg s^{-2}	E; w; q; etc.
power	Watt	W	m^2 kg s^{-3}	P
Celsius temperature	degree Celsius	°C	K - see note	$t_{°C}$
concentration	molarity	M	mol m^{-3}	C
"	mole fraction		mol mol^{-1}	X
"	molality	m	mol kg^{-1}	b
electric potential	volt	V	m^2 kg s^{-3} A^{-1}	V, ϕ
electric charge	coulomb	C	A s	Q
Volume	liter	L	dm^3	V

Note on °C: The relationship between temperature in °C, $t_{°C}$, and K, T, is:

$$T/\mathrm{K} = t_{°C}/°\mathrm{C} + 273.15 \qquad (5)$$

[#] Notice that the SI base unit for mass has a prefix, k. Grams, g, is scaled with a prefix to give increments of kilograms. For example, a thousand kilograms is written as Mg and not as kkg and is a megagram. One millionth of a kilogram is mg, or milligram, and not μkg, etc.

Quantity Calculus

The idea behind quantity calculus is that the equation 1, 6 may be treated as an ordinary algebraic equation. A common rearrangement for this equation is:

$$\textit{(physical quantity)/unit = numerical value} \qquad 6$$

Thus, equation 2 can be rewritten as:

$$\lambda/\mathrm{m} = 5.869 \times 10^{-7} \qquad 7$$

This is used in the note to the above table and equation 5. Thus, t_C/°C and T/K are numerical values. The number 273.15 in this equations is, therefore, a numerical value only; although, one could make an interpretation of its meaning.

The idea of quantity calculus leads to what is normally referred to as unit factor conversions. This is a technique to convert a physical quantity with a certain numerical value and a unit to an equivalent quantity with a different numerical value and a different unit. This is most useful for conversions between SI and the English system. It may, however, be used in any calculation where only multiplication and division is used.

For example: Convert 10.0 m to in (inches.) Give that the definition of an inch is:

$$2.54 \text{ cm} = 1 \text{ in} \quad \text{(exactly on both sides)} \qquad 8$$

Solution: To start you can either convert 10.0 m to cm using the prefix definitions, i. e.:

$$10.0 \text{ m} \equiv 10.0 \times 10^{2} \times 10^{-2} \text{ m} \equiv 10.0 \times 10^{2} \text{ cm} \equiv 1.00 \times 10^{3} \text{ cm} \qquad 9$$

or you can create the equation:

$$1 \text{ cm} \equiv 1 \times 10^{-2} \text{ m} \qquad 10$$

using the prefix definitions. For illustration, equations 8 and 10 will be used. These can be rearranged to give the following:

$$\frac{1 \text{ in}}{2.54 \text{ cm}} = 1 \qquad 11$$

and

$$\frac{1 \text{ cm}}{1 \times 10^{-2} \text{ m}} = 1 \qquad 12$$

One can now multiply the 10.0 m by 1 and leave the physical quantity unchanged. This is the general principle of unit factor. The expression on the left of equations

11, 13 and 12 are called unit factors. A physical quantity remains the true value regardless of the unit factor that one multiplies by. (This may be true, but one tries to obtain an answer which also makes some sense!) Thus in order to convert the physical quantity 10.0 m to the same physical quantity in inches, one multiplies it by the unit factor in equations 11, 13 and 12, so thus:

$$10.0\ \cancel{\text{m}} \times \frac{1\ \cancel{\text{cm}}}{1 \times 10^{-2}\ \cancel{\text{m}}} \times \frac{1\ \text{in}}{2.54\ \cancel{\text{cm}}} = 394\ \text{in} \qquad 13$$

Notice how m and cm are treated as normal algebraic quantities and are "canceled out." (That is, cm divided by cm yields unity - multiplication and division are inverses of each other.) This is merely saying, if $l = 10.0$ m then:

$$l = l \times 1 \times 1 = l\ (= 394\ \text{in}) \qquad 14$$

The numerical value changes, the unit changes, but the physical quantity does not change.

Using the quantity calculus in place of multiplicative formulas

When calculating a physical quantity, formulas are normally needed to determine an answer. For example, if the temperature is 20°C and one wishes to know the temperature in Kelvins, one uses formula 5. Substituting into this equation, one calculates a temperature of 293 K. For some formulas, one can use the principles of the unit factor method explained above to yield an answer. These formulas must have only multiplication and division and no dimensionless constants. An example of such a formula is the formula for density:

$$\rho = m/V \qquad 15$$

where ρ is density, m is mass and V is volume. The usual units used for liquids and solids are g and mL (L signifies liters $\equiv 1 \times 10^{-3}\ \text{m}^3 \approx 1$ quart) or kg and L. This formula defines the relationship between three physical quantities. One can multiply or divide two physical quantities which are known, to yield the third physical quantity. In this technique, the physical quantities given are consider "true" values and multiplying or dividing "true" values, yield other "true" values.

For example:

Aluminum has a density of 2.702 g/mL. What is the weight of an aluminum sample which has a volume of 15.0 mL.
Multiply or divide these physical quantities in such a way as to yield the physical quantity with the units of g. Or:

$$\frac{2.702\ \text{g}}{\cancel{\text{mL}}} \times 15.0\ \cancel{\text{mL}} = 40.5\ \text{g}$$